International Journal of Theoretical Physics, Vol. 27, No. 11, 1988

# $O(5)\times U(1)$ Electroweak Gauge Theory and the Triggering of the Neutrino Oscillations by 't Hooft–Polyakov Monopole

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Received March 3, 1988

The $O(5)\times U(1)$ electroweak gauge theory with two particle generations of quarks and leptons is considered. With spontaneous symmetry breaking down to the $O(3)$ level, the 't Hooft-Polyakov $SO(3)$ monopole theory along with its triplet of scalar fields is reproduced and developed to the extent necessary to establish the results. It is shown that the existence of the monopole triggers the Cabibbo rotation of $d$ and $s$ along with the $\nu_e$ and $\nu_\mu$ flavors, which in turn results in the neutrino oscillations. The neutrino oscillation angle turns out to be the Cabibbo angle. Using the experimental data of Baker *et al.*, an upper limit is set on $\Delta m^2 \leq 1.3\ \mathrm{eV}^2$ ($\Delta m^2 \equiv m_\mu^2 - m_e^2$). Furthermore, it is exactly the Cabibbo angle in which the isovector $\bar{\phi}$ has to be rotated so as to spontaneously break the symmetry down to the $O(3)$ level, together with, on the $SO(3)$ sector, to the $U(1)$ level. It turns out that the Weinberg angle is twice the Cabibbo angle, a result already noted elsewhere.

## 1. INTRODUCTION

In view of the continuing search for the magnetic monopoles, reported, e.g., in the recent work of Barish *et al.* (1987), Bartlet *et al.* (1987), and Ebisu and Watanabe (1987) and references therein, it is worthwhile searching for some theoretical evidence which could serve as a signal for the existence of the monopoles. In a previous paper (Samiullah, 1988) it was shown that the Cabibbo rotation of $d$ and $s$ quarks is triggered by the 't Hooft-Polyakov monopole given by the $SO(3)$ gauge theory. The present work indicates that the neutrino oscillations are also triggered by the $SO(3)$ monopole. It is shown that the $SO(3)$ monopole theory naturally follows from the $O(5)\times U(1)$ model and that with the explicit particle content the mere existence of a monopole triggers the Cabibbo rotation of $d$ and $s$ quarks along with the $\nu_e$ and $\nu_\mu$ neutrinos.

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0020-7748/88/1100-1355$06.00/0 

Regarding the $O(5)\times U(1)$ electroweak theory, it suffices to say that the group $O(5)$ is anomaly-free and economical in the number of gauge bosons which we associate with each of its generators. Considering that data are analyzed with the assumption that the dominant contribution comes from one pair of neutrinos (i.e., coming back to the case of two neutrino flavors) (Guyot, 1983), I employ the four-dimensional spinorial representation of the group $O(5)$ which could accommodate two of the three generations with the as yet undiscovered top quark. In Section 2, I briefly sketch the relevant mathematics. In Section 3, I discuss the $O(5)\times U(1)$ model. In Section 4 the symmetry is broken down to the $SO(3)$ level. In Section 5 the usual $SO(3)$ monopole theory is reproduced, 't Hooft's electromagnetic tensor is derived, magnetic current and charge are given, and their conservation is shown. Section 6 utilizes the conventional procedure to establish the neutrino oscillations and the oscillation length. Section 7 deals with the triggering of the neutrino oscillations by the 't Hooft–Polyakov monopole. Section 8 discusses the results.

## 2. THE RELEVANT MATHEMATICS

To establish notation and indicate the particular representation used in this work I sketch here some of the relevant mathematics. The method of constructing the spinorial representations in higher dimensions for rotation groups is discussed by Brauer and Weyl (1935). Following their method, I construct the four-dimensional representation of the group $O(5)$. Take a set of five $4\times 4$ Hermitian anticommuting matrices $\Gamma_a$:

$$\Gamma_a^\dagger=\Gamma_a, \qquad \{\Gamma_a,\Gamma_b\}=2\delta_{ab}, \qquad a,b=1,\ldots,5 \tag{1}$$

and

$$\begin{aligned}\Gamma_1&=\sigma_1^{(1)}\times\sigma_1^{(2)}, \qquad \Gamma_2=\sigma_1^{(1)}\times\sigma_2^{(2)}, \qquad \Gamma_3=\sigma_3^{(1)}\times 1\\ \Gamma_4&=\sigma_1^{(1)}\times\sigma_3^{(2)}, \qquad \Gamma_5=\sigma_2^{(1)}\times 1\end{aligned} \tag{2}$$

The superscripts (1) and (2) refer to two distinct sets of Pauli matrices, the symbol $\times$ stands for the direct product, and 1 stands for the $2\times 2$ unit matrix.

The generators are given by

$$F_{ab}=-\tfrac{1}{2}i\Gamma_a\Gamma_b, \qquad a\neq b \tag{3}$$

The restriction is imposed due to the antisymmetry of $F_{ab}$. Explicitly written out, the matrices read

$$\begin{aligned}&\Gamma_1=\begin{pmatrix}0&\sigma_1\\ \sigma_1&0\end{pmatrix}, \qquad \Gamma_2=\begin{pmatrix}0&\sigma_2\\ \sigma_2&0\end{pmatrix}, \qquad \Gamma_3=\begin{pmatrix}1&0\\ 0&-1\end{pmatrix}\\ &\Gamma_4=\begin{pmatrix}0&\sigma_3\\ \sigma_3&0\end{pmatrix}, \qquad \Gamma_5=\begin{pmatrix}0&-i\times 1\\ i\times 1&0\end{pmatrix}\end{aligned} \tag{4}$$

The generators are given as follows:

$$
\begin{aligned}
&F_{12}=\frac{1}{2}\begin{pmatrix}\sigma_3 & 0\\ 0 & \sigma_3\end{pmatrix}, \quad F_{13}=\frac{1}{2}\begin{pmatrix}0 & i\sigma_1\\ -i\sigma_1 & 0\end{pmatrix}, \quad F_{14}=\frac{1}{2}\begin{pmatrix}-\sigma_2 & 0\\ 0 & -\sigma_2\end{pmatrix}\\
&F_{15}=\frac{1}{2}\begin{pmatrix}\sigma_1 & 0\\ 0 & -\sigma_1\end{pmatrix}, \quad F_{23}=\frac{1}{2}\begin{pmatrix}0 & i\sigma_2\\ -i\sigma_2 & 0\end{pmatrix}, \quad F_{24}=\frac{1}{2}\begin{pmatrix}\sigma_1 & 0\\ 0 & \sigma_1\end{pmatrix}\\
&F_{25}=\frac{1}{2}\begin{pmatrix}\sigma_2 & 0\\ 0 & -\sigma_2\end{pmatrix}, \quad F_{34}=\frac{1}{2}\begin{pmatrix}0 & -i\sigma_3\\ i\sigma_3 & 0\end{pmatrix}, \quad F_{35}=\frac{1}{2}\begin{pmatrix}0 & -1\\ -1 & 0\end{pmatrix}\\
&F_{45}=\frac{1}{2}\begin{pmatrix}\sigma_3 & 0\\ 0 & -\sigma_3\end{pmatrix}
\end{aligned}
\tag{5}
$$

They satisfy the following commutation relation:

$$[F_{ab}, F_{cd}] = i(\delta_{ac}F_{bd} - \delta_{bc}F_{ad} + \delta_{bd}F_{ac} - \delta_{ad}F_{bc}) \tag{6}$$

forming a Lie algebra.

It is convenient to consider the algebra in a different basis

$$\{F_i, F_{45}, F_i^{\pm}\}, \qquad i=1,2,3 \tag{7}$$

defined as follows:

$$F_1 = F_{23}, \qquad F_2 = F_{13}, \qquad F_3 = F_{12} \tag{8}$$

$$F_1^{\pm} = F_{14} \pm iF_{15}, \qquad F_2^{\pm} = F_{24} \pm iF_{25}, \qquad F_3^{\pm} = F_{34} \pm iF_{35} \tag{9}$$

Among the above set of generators using equation (6), in particular, the following commutation relations can be established:

$$
\begin{aligned}
&[F_i^{\pm}, F_i^{\mp}] = \pm 2F_{45} \qquad (i \text{ not summed})\\
&[F_{45}, F_i^{\pm}] = \pm F_i^{\pm}
\end{aligned}
\tag{10}
$$

$$[F_{45}, F_i] = 0 \tag{11}$$

$$[F_i, F_j] = i\varepsilon_{ijk}F_k \tag{12}$$

From equation (10) we see that for every value of $i$ $(=1,2,3)$ the set of generators $\{F_{45}, F_2^{\pm}\}$ and, from equation (12), the other one, i.e., $\{F_i\}$, form $su(2)$ subalgebras. Since in equations (13) the charge operator is defined using the generator $F_{45}$, equations (10) and (11) indicate that $F_i$ and $F_i^{\pm}$ are the eigenvectors of the charge operator with the eigenvalues 0 and $\pm 1$, respectively, the charge is invariant under the group $O(5)$, and eventually under the larger group $O(5)\times U(1)$.

## 3. THE $O(5)\times U(1)$ MODEL

The present work develops the electroweak theory using the four-dimensional spinorial representation of the group $O(5)$ to which the left-handed quark and lepton multiplets $Q_L^T = (u, d, s, c)_L$ and $L_L^T = (\nu_e, e, \mu, \nu_\mu)_L$ are assigned, whereas the right-handed particles $u_R$, $d_R$, $s_R$,

$c_R$, $e_R$, $\mu_R$ are taken to be the singlets of the group. The model has three sets of gauge bosons: (1) analogs of the Glashow (1961), Weinberg (1967), and Salam (1968) (GWS) model, (2) additional charged gauge bosons as compared to the GWS model, and (3) a set of three neutral gauge bosons.

This model has ten gauge fields $W_{ij}$ $(i<j=1,\ldots,5)$ and a singlet one transforming as $O(5)$ and $U(1)$ generators, respectively. Table I gives the eigenvalues of the operators $F_{45}$ and $F_0$ along with their charges for the leptons and quarks.

The eigenvalues $Y_{45}$ of the operator $F_{45}$ for the right-handed particles $u_R$, $d_R$, $s_R$, $c_R$, $e_R$, $\mu_R$ are taken to be zero, as they are the singlets and do not belong to the four-dimensional representation of the group $O(5)$. In terms of the $O(5)\times U(1)$ generators, the charge operator is given as

$$Q=F_{45}+\tfrac{1}{2}F_0 \tag{13}$$

Because of equations (8) and (9), it is possible to define a basis for the gauge bosons such that in the Lagrangian (19) certain combinations of the gauge fields, for instance $(1/\sqrt{2})(W_\mu^{24}+iW_\mu^{25})$, can be universally coupled to the charge currents such as $u\gamma^\mu(1/2)(1+\gamma^5)d$, $c\gamma^\mu(1/2)(1+\gamma^5)s$ rather than the separate ones $W_\mu^{24}$ and $W_\mu^{25}$.

We define

$$\begin{gathered}F_C=F_{12},\qquad F_D=F_{13},\qquad F_E=F_{23},\qquad F_F=F_{45}\\ F_U^\pm=\frac{1}{\sqrt{2}}F_1^\pm,\qquad F_V^\pm=\frac{1}{\sqrt{2}}F_3^\pm,\qquad F_W^\pm=\frac{1}{\sqrt{2}}F_2^\pm\end{gathered} \tag{14}$$

The corresponding basis for the gauge fields is taken to be

$$\begin{gathered}C_\mu=W_\mu^{12},\qquad D_\mu=W_\mu^{13},\qquad E_\mu=W_\mu^{23},\qquad F_\mu=W_\mu^{25}\\ U_\mu^\pm=\pm\frac{i}{\sqrt{2}}(W_\mu^{14}\mp iW_\mu^{15}),\qquad V_\mu^\pm=\mp\frac{i}{\sqrt{2}}(W_\mu^{34}\mp iW_\mu^{35})\\ W_\mu^\pm=\frac{1}{\sqrt{2}}(W_\mu^{24}\mp iW_\mu^{25})\end{gathered} \tag{15}$$

**Table I**

| | Leptons | | | | Quarks | | | |
|---|---|---|---|---|---|---|---|---|
| | $\nu_{e,L}$ | $e_{L,R}$ | $\mu_{L,R}$ | $\nu_{\mu,L}$ | $u_{L,R}$ | $d_{L,R}$ | $s_{L,R}$ | $c_{L,R}$ |
| $Q$ | 0 | −1 | −1 | 0 | 2/3 | −1/3 | −1/3 | 2/3 |
| $Y_{45}$ | 1/2 | −1/2, 0 | −1/2, 0 | 1/2 | 1/2, 0 | −1/2, 0 | −1/2, 0 | 1/2, 0 |
| $Y_0$ | −1 | −1, −2 | −1, −2 | −1 | 1/3, 4/3 | 1/3, −2/3 | 1/3, −2/3 | 1/3, 4/3 |

Denoting the gauge couplings for the groups $O(5)$ by $g$ and for $U(1)$ by $(1/2)g'$, we can express the coupling of the fermion currents ($\psi$ representing both the quark and the lepton fields) to the gauge bosons [with $\bar{a}$ taken to be the Dirac conjugate of $a$ and introducing the abbreviations $a_L = (1/2)(1+\gamma^5)a$, $a_R = (1/2)(1-\gamma^5)a$, $\bar{a}\gamma^\mu b \to \bar{a}\gamma^\mu(1/2)(1+\gamma^5)b$] by the following interaction Lagrangian

$$
\begin{aligned}
L_{\text{int}} = g \sum_{i<j} & (\bar{\psi}_L \gamma^\mu F_{ij} W^{ij}_\mu \psi_L) \\
& + \tfrac{1}{2} g' [\{\tfrac{1}{3}(\bar{u}_L \gamma^\mu W^0_\mu u_L + \bar{d}_L \gamma^\mu W^0_\mu d_L + \bar{s}_L \gamma^\mu W^0_\mu s_L + \bar{c}_L \gamma^\mu W^0_\mu c_L) \\
& - (\bar{\nu}_{eL} \gamma^\mu \nu_{eL} + \bar{e}_L \gamma^\mu W^0_\mu e_L + \bar{\mu}_L \gamma^\mu W^0_\mu \mu_L + \bar{\nu}_{\mu L} \gamma^\mu W^0_\mu \nu_{\mu L})\} \\
& + \{(\bar{u}_L \gamma^\mu \tfrac{4}{3} W^0_\mu u_L - \bar{d}_L \gamma^\mu \tfrac{2}{3} d_L - \bar{s}_L \gamma^\mu \tfrac{2}{3} W^0_\mu s_L + \bar{c}_L \gamma^\mu \tfrac{4}{3} W^0_\mu c_L) \\
& - 2(\bar{e}_R \gamma^\mu W^0_\mu e_R + \bar{\mu}_R \gamma^\mu W^0_\mu \mu_R)\}]
\end{aligned}
\tag{16}
$$

Furthermore, defining

$$
J_\mu(em) = (\tfrac{2}{3}\bar{u}\gamma^\mu u - \tfrac{1}{3}\bar{d}\gamma^\mu d - \tfrac{1}{3}\bar{s}\gamma^\mu s + \tfrac{2}{3}\bar{c}\gamma^\mu c - \bar{e}\gamma^\mu e - \bar{\mu}\gamma^\mu \mu) \tag{17}
$$

$$
\begin{aligned}
J_{\mu,l} = (& \bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L - \bar{s}_L \gamma^\mu s_L + \bar{c}_L \gamma^\mu c_L \\
& - \bar{e}_L \gamma^\mu e_L - \bar{\mu}_L \gamma^\mu \mu_L + \bar{\nu}_{eL} \gamma^\mu \nu_{eL} + \bar{\nu}_\mu \gamma^\mu \nu_{\mu L})
\end{aligned}
\tag{18}
$$

the interaction Lagrangian can be recast as follows:

$$
\begin{aligned}
L_{\text{int}} = g' W^0_\mu J_\mu(em) & + \tfrac{1}{2}(gF_\mu - g' W^0_\mu) J_{\mu,l} \\
& + \tfrac{1}{2} g c_\mu (\bar{u}_L \gamma^\mu u_L - \bar{d}_L \gamma^\mu d_L + \bar{s}_L \gamma^\mu s_L - \bar{c}_L \gamma^\mu c_L \\
& - \bar{e}_L \gamma^\mu e_L + \bar{\mu}_L \gamma^\mu \mu_L - \bar{\nu}_{\mu L} \gamma^\mu \nu_{\mu L} + \bar{\nu}_{eL} \gamma^\mu \nu_{eL}) \\
& + \tfrac{1}{2} g i D_\mu (\bar{u}_L \gamma^\mu c_L + \bar{d}_L \gamma^\mu s_L - \bar{s}_L \gamma^\mu d_L - \bar{c}_L \gamma^\mu u_L + \bar{\nu}_{eL} \gamma^\mu \nu_{\mu L} \\
& + \bar{e}_L \gamma^\mu \mu_L - \bar{\mu}_L \gamma^\mu e_L - \bar{\nu}_{\mu L} \gamma^\mu \nu_{eL} + \bar{\nu}_{eL} \gamma^\mu \nu_{\mu L}) \\
& + \tfrac{1}{2} g E_\mu (\bar{u}_L \gamma^\mu c_L - \bar{d}_L \gamma^\mu s_L - \bar{s}_L \gamma^\mu d_L - \bar{c}_L \gamma^\mu u_L \\
& + \bar{\nu}_{eL} \gamma^\mu \nu_{eL} - \bar{e}_L \gamma^\mu \mu_L - \bar{\mu}_L \gamma^\mu e_L + \bar{\nu}_{\mu L} \gamma^\mu \nu_{eL})
\end{aligned}
\tag{19}
$$

At this stage it is relevant to mention that we work here, conventionally, in the Prasad–Sommerfield (1975) limit, in which we do not give the potential explicitly but only specify that it gives to the scalar fields in the ground state a nonvanishing vacuum expectation value. The nature of the vacuum expectation value determines the unbroken subgroup of the bigger one.

## 4. THE SYMMETRY BREAKING

To reproduce the $SO(3)$ theory of the magnetic monopole, we break the $O(5)\times U(1)$ symmetry spontaneously down to the $O(3)$ level in two stages. First we break the symmetry to the $O(4)$ level by employing a complex scalar field $\phi=\phi^b$ and its complex conjugate $\phi^*=\phi^c$ and then a real scalar field $\phi^a$ (Higgs) to the $O(3)$ level.

The real Higgs coupling is given by Lie (1974) as

$$L_W\phi^a=\tfrac{1}{2}(\partial_\mu\phi_i^a-gW_{uik}\phi_k^a)(\partial_\mu\phi_i^a-gW_{\mu il}\phi_l^a) \tag{20}$$

The complex Higgs are coupled in the following way:

$$L_W\phi_{\text{complex}}=\tfrac{1}{2}(\partial_\mu\phi_i-gW_{\mu ik}\phi_k-ig'/2\,W_\mu^0\phi_i)^\dagger \times(\partial_\mu\phi_i-gW_{\mu il}\phi_l-ig'/2\,W_\mu^0\phi_i) \tag{21}$$

We choose the following ground-state expectation values

$$\langle\phi^a\rangle=\delta_{i4}V_4,\qquad \langle\phi^b\rangle=\langle\phi^c\rangle=\delta_{i5}V_5 \tag{22}$$

where $V_4$ and $V_5$ are constants.

Substituting the expectation values of $\phi^b$ and $\phi^c$ in equation (21), we find

$$V_5^2g^2(|W_{\mu 15}|^2+|W_{\mu 25}|^2+|W_{\mu 35}|^2+|W_{\mu 45}|^2)+V_5^2g'^2/4\,W_\mu^{02} \tag{23}$$

On the other hand, substituting the expectation value of $\phi^a$ in equation (20), we get

$$V_4^2g^2(|W_{\mu 14}|^2+|W_{\mu 24}|^2+|W_{\mu 35}|^2+|W_{\mu 45}|^2) \tag{24}$$

With $V_4=V_5=V$, adding the relations (23) and (24) and using the definitions (15), after the first and the second stage of symmetry breaking, the boson mass term is given as

$$\tfrac{1}{2}g^2V^2(U_\mu^+U_\mu^-+V_\mu^+V_\mu^-+W_\mu^+W_\mu^-+\tfrac{1}{4}F_\mu^2+g'^2/g^2\,W_\mu^{02}) \tag{25}$$

From equation (25) we notice that $U_\mu^\pm$, $V_\mu^\pm$, $W_\mu^\pm$, $F_\mu$ and $W_\mu^0$ have acquired masses. Since the generators $F_{12}$, $F_{13}$ and $F_{23}$ remain unbroken as such, the corresponding particles $C_\mu$, $D_\mu$ and $E_\mu$ are massless; we also notice that they are neutral. From (12) we learn that the generators $F_{12}$, $F_{13}$ and $F_{23}$ form an $O(3)$ group and hence the symmetry is reduced to the $O(3)$ level. Now our theory contains an $SO(3)$ group and a triplet of Higgs scalars $\phi^a$, $\phi^b$ and $\phi^c$. This scenario is exactly the starting point of the 't Hooft–Polyakov monopole theory; the difference is that at the $SO(3)$ sector the present theory employs the five-dimensional representation, in contrast to the usual three-dimensional one.

## 5. THE $SO(3)$ MONOPOLE THEORY

At the $SO(3)$ sector equipped with a triplet of Higgs scalars we can write the relevant Lagrangian as follows:

$$L=-\tfrac{1}{4}\bar{G}^{\mu\nu}\cdot\bar{G}_{\mu\nu}+\tfrac{1}{2}D^{\mu}\bar{\phi}\cdot D_{\mu}\bar{\phi}-V(\bar{\phi}) \qquad (26)$$

where $G_{\mu\nu}$ is the gauge-invariant tensor, for the non-Abelian group $SO(3)$; it is defined by

$$\bar{G}_{\mu\nu}=\partial_{\mu}\bar{W}_{\nu}-\partial_{\nu}\bar{W}_{\mu}+g\bar{W}_{\mu}\times\bar{W}_{\nu} \qquad (27)$$

and $\bar{\phi}$ has a covariant derivative given by

$$D_{\mu}\bar{\phi}=\partial_{\mu}\bar{\phi}+g\bar{W}_{\mu}\times\bar{\phi} \qquad (28)$$

its components being $\phi^{a}$, $\phi^{b}$ and $\phi^{c}$.

We now specify the potential as

$$V(\bar{\phi})=-\tfrac{1}{4}\lambda(\bar{\phi}^{2}-a^{2}) \qquad (29)$$

which, at $r\gg a$, in Higgs vacuum, defined in equations (34) and (35) yields $\bar{\phi}^{2}=a^{2}$. For this sector the equations of motion are given as follows:

$$D_{\nu}\bar{G}^{\mu\nu}=g\bar{\phi}\times D^{\mu}\bar{\phi} \qquad (30)$$

$$D_{\mu}D^{\mu}\bar{\phi}=-\lambda\bar{\phi}(\bar{\phi}^{2}-a^{2}) \qquad (31)$$

The above equations in component form read

$$(D_{\nu}G^{\mu\nu})_{i}=-g\varepsilon_{ijk}\phi_{k}(D^{\mu}\phi)_{k} \qquad (32)$$

$$(D^{\mu}D_{\mu}\phi)_{i}=-\lambda\phi_{i}(\bar{\phi}^{2}-a^{2}) \qquad (33)$$

We say that the fields in a certain region of space-time are in the Higgs vacuum if the equations

$$(G^{\mu\nu}\phi)_{i}=0 \qquad (34)$$

$$(D^{\mu}\phi)_{i}=0 \qquad (35)$$

are satisfied. Furthermore, we make the conventional assumption that any finite-energy solution, except in a finite number of compact localized regions which we associate with the magnetic monopoles, at large distances, satisfies equations (34) and (35). For a given $\bar{\phi}$, outside the monopole regions in the Higgs vacuum the general form of $W_{\mu}$ is given by Corrigan *et al.* (1976) as

$$\bar{W}^{\mu}=\frac{1}{a^{2}g}\bar{\phi}\times\partial^{\mu}\bar{\phi}+\frac{1}{a}\bar{\phi}A^{\mu} \qquad (36)$$

where $A^\mu$ is arbitrary. From (36) with $\bar{\phi}/a=\hat{\phi}$ we find

$$A^\mu=\hat{\phi}\cdot\bar{W}^\mu=\hat{\phi}^i W^i{}_\mu \tag{37}$$

Defining

$$F^{\mu\nu}=\frac{1}{a^3 g}\bar{\phi}\cdot(\partial^\mu\bar{\phi}\times\partial^\nu\bar{\phi})+\partial^\mu A^\nu-\partial^\nu A^\mu \tag{38}$$

it can be easily checked that the following equation is derivable:

$$\bar{G}^{\mu\nu}=\frac{1}{a}\bar{\phi}F^{\mu\nu} \tag{39}$$

or equivalently one gets

$$F^{\mu\nu}=\frac{\bar{\phi}}{a}\cdot\bar{G}^{\mu\nu} \tag{40}$$

From equation (28) we note that in the direction where $\bar{\phi}$ obtains a constant value in the Higgs vacuum we get

$$\partial_\nu F^{\mu\nu}=0 \qquad \text{and} \qquad \partial_\nu {}^*F^{\mu\nu}=0 \tag{41}$$

where ${}^*F^{\mu\nu}$ is the dual of the field tensor $F^{\mu\nu}$ defined as

$${}^*F^{\mu\nu}=1/2\varepsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

$\varepsilon^{\mu\nu\rho\sigma}$ being the completely antisymmetric unit tensor. With

$$F^{0i}=-E^i \qquad \text{and} \qquad F^{ij}=-\varepsilon_{ijk}B^k \tag{42}$$

where $\bar{E}$ and $\bar{B}$ are the electric and magnetic fields, respectively, we recognize (41) as the Maxwell equations. Defining $\bar{\phi}/a=\hat{\phi}$, one can reexpress equation (38) as

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu+\frac{1}{g}\hat{\phi}\cdot(\partial^\mu\hat{\phi}\times\partial^\nu\hat{\phi})=\hat{\phi}\cdot\bar{G}^{\mu\nu} \tag{43}$$

which in component form becomes

$$F^{\mu\nu}=\partial^\mu A^\nu-\partial^\nu A^\mu+\frac{1}{g}\varepsilon_{ijk}\hat{\phi}^i\,\partial^\mu\hat{\phi}^j\,\partial^\nu\hat{\phi}^k \tag{44}$$

Equation (44) is 't Hooft's "electromagnetic" tensor. It may be remarked that the field equations (32) and (33) are second-order nonlinear coupled partial differential equations and finding a general solution of these equations is an extremely difficult task. However, as an example of its usefulness, 't Hooft circumvented the difficulty of solving these equations by employing a static, spherically symmetric ansatz, namely

$$\begin{aligned}(W_0)_a&=0, \qquad (W_i)_a=\varepsilon_{aij}x^j[1-K(r)]/gr^2\\ \hat{\phi}^a&=x^a H(r)/gr^2, \qquad r^2=x_1^2+x_2^2+x_3^2\end{aligned} \tag{45}$$

In the Prasad-Sommerfield limit, equation (27) has an explicit solution (see, for example, Marciano, 1978):

$$K(r)=gar/\sinh(gar);\qquad H(r)=gar\coth(gar)-1 \tag{46}$$

Equations (44)-(46) lead to

$$F_{ij}=-\varepsilon_{ijk}x^k/gr^3 \tag{47}$$

which corresponds to the magnetic field of a point monopole with the magnetic charge $1/g$.

The topological nature of this magnetic charge was demonstrated by Arafune *et al.* (1975). From equation (44) they noted that if $A_\mu$ is free of string singularities, one can define the magnetic current as

$$^*K_\mu=\partial^\nu\,{}^*F_{\mu\nu}=\frac{1}{2g}\varepsilon_{\mu\nu\rho\sigma}\varepsilon_{abc}\,\partial^\nu\hat{\phi}^a\,\partial^\rho\hat{\phi}^b\,\partial^\sigma\hat{\phi}^\sigma \tag{48}$$

This current is conserved

$$\partial^\mu\,{}^*K_\mu=0 \tag{49}$$

The corresponding charge

$$M=\frac{1}{4\pi}\int K_0\,d^3x=\frac{1}{g}$$

yields

$$\dot{M}=0 \tag{50}$$

and hence it is also conserved.

## 6. THE NEUTRINO OSCILLATIONS

To put the present work in the proper perspective, I briefly recall here some of the relevant work on neutrino oscillations (for example, see Commins and Bucksbaum, 1983). Define the states $\nu_{e\theta}$ and $\nu_{\mu\theta}$ as the following superposition of the neutrinos $\nu_e$ and $\nu_\mu$:

$$\begin{aligned}\nu_{e\theta}&=\cos\theta\,\nu_e+\sin\theta\,\nu_\mu\\ \nu_\mu\theta&=-\sin\theta\,\nu_e+\cos\theta\,\nu_\mu\end{aligned} \tag{51}$$

Consider that the neutrinos $\nu_e$ and $\nu_\mu$ are formed at time $t=0$ in the states $\nu_e(0)$ and $\nu_\mu(0)$, respectively, and have the time evolution given by

$$\begin{aligned}|\nu_e(t)\rangle&=\exp(-iE\nu_e t/\hbar)|\nu_e(0)\rangle\\ |\nu_\mu(t)\rangle&=\exp(-iE\nu_\mu t/\hbar)|\nu_\mu(0)\rangle\end{aligned} \tag{52}$$

With $P_{\nu_e} = P_{\nu_\mu} = P$, we have

$$E_{\nu_e} = (P_{\nu_e}^2 + m_{\nu_e}^2)^{1/2} = (P^2 + m_{\nu_e}^2)^{1/2}$$
$$E_{\nu_\mu} = (P_{\nu_\mu}^2 + m_{\nu_\mu}^2)^{1/2} = (P^2 + m_{\nu_\mu}^2)^{1/2} \tag{53}$$

Equation (51) becomes

$$|\nu_{e\theta}(t)\rangle = \exp(-iE_{\nu_e}t/\hbar)|\nu_e(0)\rangle \cos\theta + \exp(-iE_{\nu_\mu}t/\hbar)|\nu_\mu(0)\rangle \sin\theta \tag{54}$$

This can be written as

$$|\nu_{e\theta}(t)\rangle = \{\exp(-iE_{\nu_e}t/\hbar)\cos^2\theta + \exp(-iE_{\nu_\mu}t/\hbar)\sin^2\theta\}|\nu_{e\theta}(0)\rangle$$
$$+ \cos\theta \sin\theta\{-iE_{\nu_\mu}t/\hbar) - \exp(-iE_{\nu_e}t/\hbar)\}|\nu_{\mu\theta}(0)\rangle \tag{55}$$

The probability for a neutrino originally in the state $|\nu_{e\theta}\rangle$ to be in the state $|\nu_{\mu\theta}\rangle$ at time $t$ is given by

$$P(\nu_{e\theta} \leftrightarrow \nu_{\mu\theta}) = |\langle\nu_\mu|\nu_e(t)\rangle|^2$$
$$= \cos^2\theta \sin^2\theta\, |\exp(-iE_{\nu_\mu}t/\hbar) - \exp(-iE_{\nu_e}t/\hbar)|^2$$
$$= \tfrac{1}{2}\sin^2 2\theta\, \{1 - \cos\theta\, (E_{\nu_\mu} - E_{\nu_e})t/\hbar\} \tag{56}$$

If a neutrino beam travels a distance $R$ in time $t$, putting $R = ct$ and assuming $P \gg m_{\nu_e}, m_{\nu_\mu}$, we can rewrite equation (56) as

$$P(\nu_{e\theta} \leftrightarrow \nu_{\mu\theta}) = \frac{1}{2}\sin^2 2\theta \left(1 - \cos\theta \frac{m_{\nu_\mu}^2 - m_{\nu_e}^2}{2p}\frac{c^2}{\hbar}R\right) \tag{57}$$

The neutrino oscillation length is defined by the equation

$$\frac{m_{\nu_\mu}^2 - m_{\nu_e}^2}{2p}\frac{c^2}{\hbar}R = \frac{2\pi R}{L} \tag{58}$$

where $L$ is the oscillation length; equation (58) yields

$$L = \frac{4\pi\hbar p}{(m_{\nu_\mu}^2 - m_{\nu_e}^2)c^2} \tag{59}$$

As already remarked in Section 1, in view of the fact that usually three neutrino flavors data are analyzed assuming that the dominant contribution comes only from one pair of neutrinos, even a detailed calculation would lead to the same result as given in equations (57) and (59).

## 7. TRIGGERING OF THE NEUTRINO OSCILLATIONS

Inserting (37) in (38) in a gauge where $\bar{\phi}$ assumes a constant value, we obtain

$$F_{\mu\nu} = \partial_\mu(\hat{\phi}^i W_\nu^i) - \partial_\nu(\hat{\phi}^i W_\mu^i) \tag{60}$$

From equation (60) we notice that the gauge fields $W_\mu^{12}$, $W_\mu^{13}$ and $W_\mu^{23}$, which in view of equation (12) transform as a $SO(3)$ isovector, are rotated in the direction of $\hat{\phi}$, where it has a constant value. On the other hand, we have already remarked that when $\bar{\phi}$ takes a constant value, it is in the Higgs vacuum and breaks the $O(5)\times U(1)$ symmetry spontaneously down to the $SO(3)$ level. Furthermore, from equation (60) we notice that $\hat{\phi}$, at the $SO(3)$ sector, also breaks the symmetry down to $U(1)$. Consistency demands that any other structures existing in the theory, such as isodoublets, isotriplets, etc., should be rotated appropriately.

A $2\times 2$ unitary unimodular matrix corresponding to a three-dimensional rotation by an angle $2\theta$ around a vector $\hat{n}$ with $\bar{\sigma}$ being the Pauli matrices is given by

$$U_R = \exp\left(\frac{i}{2}2\theta\hat{n}\cdot\bar{\sigma}\right) \tag{61}$$

The set of all such $2\times 2$ matrices forms a $SU(2)$ subgroup. For simplicity, if we choose the axis of rotation as the $y$ axis, equation (61) yields the matrix

$$U_R = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \tag{62}$$

To a unitary matrix $U\in SU(2)$ there corresponds in three dimensions a unique rotation matrix $R(U)\in SO(3)$ given by the relation

$$R_{ij}(U) = \tfrac{1}{2}\mathrm{Tr}(U^\dagger\sigma_i U\sigma_j) \tag{63}$$

Explicitly written out, it reads

$$\begin{pmatrix} \cos 2\theta & 0 & -\sin 2\theta \\ 0 & 1 & 0 \\ \sin 2\theta & 0 & \cos 2\theta \end{pmatrix} \tag{64}$$

Whence to every set $\{A, B\}$ of matrices $A\in SU(2)$ and $B\in SU(2)$ belonging to two different groups there exists a definite transformation $\Lambda\in SO(4)$ satisfying the relation

$$\Lambda_{\alpha\beta} = \tfrac{1}{2}\mathrm{Tr}(\tau_\alpha A\tau_\beta B^\dagger) \tag{65}$$

where

$$\tau_\alpha = (1, i\sigma_i) \qquad \text{for} \quad \alpha = (0, i) \tag{66}$$

$$A = B = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{67}$$

$$\Lambda_{\alpha\beta} = \begin{pmatrix} \cos 2\theta & 0 & -\sin 2\theta & 0 \\ 0 & 1 & 0 & 0 \\ \sin 2\theta & 0 & \cos 2\theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{68}$$

However, in the interaction Lagrangian (19), we have, apart from $C_\mu$, $D_\mu$ and $E_\mu$, further sets of gauge bosons, namely $U^\pm_\mu$, $V^\pm_\mu$, and $W^\pm_\mu$, $W^0_\mu$, $F_\mu$. The set $\{C_\mu, D_\mu, E_\mu\}$ being an isovector, its components transform, operated upon by the matrix (55) with $\tilde{C}_\mu$, $\tilde{D}_\mu$ and $\tilde{E}_\mu$ regarded as physical gauge bosons, as

$$\begin{aligned} C_\mu &= \cos 2\theta\, \tilde{C}_\mu - \sin 2\theta\, \tilde{E}_\mu \\ D_\mu &= \tilde{D}_\mu \\ E_\mu &= -\sin 2\theta\, \tilde{C}_\mu + \cos 2\theta\, \tilde{E}_\mu \end{aligned} \tag{69}$$

Substituting equation (55) in $L_{\text{int}}$ and sticking to the usual convention of rotating the $d$ and $s$ quarks together with rotation of the $\nu_e$ and $\nu_\mu$ neutrinos, we obtain for the terms containing $C_\mu$, $D_\mu$ and $E_\mu$ the following expression:

$$\begin{aligned} &\tfrac{1}{2}g\tilde{C}_\mu[\bar{u}\gamma^\mu u + (\bar{s}\gamma^\mu s - \bar{d}\gamma^\mu d)\cos 2\theta \\ &\quad -(\bar{d}\gamma^\mu s + \bar{s}\gamma^\mu d)\sin 2\theta + \bar{\mu}\gamma^\mu\mu + (\bar{\nu}_e\gamma^\mu\nu_\mu + \bar{\nu}_\mu\gamma^\mu\nu_e)\sin 2\theta \\ &\quad +(\bar{\nu}_e\gamma^\mu\nu_e - \bar{\nu}_\mu\gamma^\mu\nu_\mu)\cos 2\theta - \bar{e}\gamma^\mu e]_L + \tfrac{1}{2}gi\tilde{D}_\mu(\bar{u}\gamma^\mu c + \bar{d}\gamma^\mu s - \bar{s}\gamma^\mu d \\ &\quad -\bar{c}\gamma^\mu u + \bar{\nu}_e\gamma^\mu\nu_\mu + \bar{e}\gamma^\mu\mu - \bar{\mu}\gamma^\mu e - \bar{\nu}_\mu\gamma^\nu\nu_e)_L \\ &\quad +\tfrac{1}{2}g\tilde{E}_\mu[\bar{u}\gamma^\mu c + (\bar{d}\gamma^\mu d - \bar{s}\gamma^\mu s)\sin 2\theta \\ &\quad -(\bar{d}\gamma^\mu s + \bar{s}\gamma^\mu d)\cos 2\theta + \bar{c}\gamma^\mu u + (\bar{\nu}_\mu\gamma^\mu\nu_e + \bar{\nu}_e\gamma^\mu\nu_\mu)\cos 2\theta \\ &\quad +(\bar{\nu}_\mu\gamma^\mu\nu_\mu - \bar{\nu}_e\gamma^\mu\nu_e)\sin 2\theta - \bar{e}\gamma^\mu\mu - \bar{\mu}\gamma^\mu e]_L \end{aligned} \tag{70}$$

If we identify $\theta$ with the Cabibbo angle $\theta_c$, relation (61) is exactly the expression one would obtain for this part of the $L_{\text{int}}$ by introducing the Cabibbo rotation of $d$, $s$; $\nu_e$, $\nu_\mu$ quarks and neutrinos given as follows:

$$d_c = \cos\theta\, d + \sin\theta\, s \qquad \nu_{e\theta} = \cos\theta\, \nu_c + \sin\theta\nu_\mu \tag{71}$$

$$s_c = -\sin\theta\, d + \cos\theta\, s \qquad \nu_{\mu\theta} = -\sin\theta\, \nu_e + \cos\theta\, \nu_\mu \tag{72}$$

At this juncture I note that regarding $\tilde{C}_\mu$, $\tilde{D}_\mu$ and $\tilde{E}_\mu$ as physical gauge bosons automatically kept the expression coupled to $C_\mu$, $D_\mu$ and $E_\mu$ in $L_{\text{int}}$ invariant. The set of particles $W^+_\mu$, $W^-_\mu$, $F_\mu$ transform as the generators $(1/\sqrt{2})F^+_2$, $(1/\sqrt{2})F^-_2$, $F_{45}$, whereas the set $W^+_\mu$, $W^-_\mu$, $F_0$ transform as the generators $(1/\sqrt{2})F^+_2$, $(1/\sqrt{2})F^-_2$, $F_0$. From equation (5) these generators transform as $SO(3)$ vectors with equal norms, i.e., (3/4)I. Hence, recalling our choice of the rotation axis to be the $y$ axis, the corresponding $SU(2)$ matrices say $A$ and $B$ turn out to be as follows:

$$A = B = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{73}$$

Thus, applying equation (69) to the set $\{W^+_\mu, W^-_\mu, F_\mu, W^0_\mu\}$, it can be reexpressed in terms of the transformed $\tilde{W}^+_\mu$, $\tilde{W}^-_\mu$, $\tilde{F}_\mu$, $\tilde{W}^0_\mu$ as follows:

$$\begin{aligned} W^\pm_\mu &= \tilde{W}^\pm_\mu \\ W^0_\mu &= \cos 2\theta\, \tilde{W}^0_\mu - \sin 2\theta\, \tilde{F}_\mu \\ F_\mu &= \sin 2\theta\, \tilde{W}^0_\mu + \cos 2\theta\, \tilde{F}_\mu \end{aligned} \tag{74}$$

Using (74), it is possible to rewrite the part of the $L_{\text{int}}$

$$g' W^0_\mu J_\mu(em) + \tfrac{1}{2}(gF_\mu - g' W^0_\mu) J_{\mu,l} \tag{75}$$

as follows:

$$\frac{gg'}{(g^2+g'^2)^{1/2}} \tilde{W}^0_\mu J_\mu(em) + \frac{\tilde{F}_{\mu,l}}{(g^2+g'^2)^{1/2}} [\tfrac{1}{2}(g^2+g'^2) J_{\mu,l} - g'^2 J_\mu(em)] \tag{76}$$

where we have defined

$$\tan 2\theta = g'/g \tag{77}$$

It is interesting to note that if we identify $gg'/(g^2+g'^2)^{1/2}$ as usual with the charge $e$ and rewrite

$$2\theta = \theta_W \tag{78}$$

one can identify $\tilde{W}^0_\mu$ with $A_\mu$ and $\tilde{F}_{\mu,l}$ with $Z^0_\mu$, and equation (76) is exactly the expression we would obtain if we Cabibbo-rotate the quark $d$, $s$ and the $\nu_e$ and $\nu_\mu$ neutrinos. Hence, $\theta$ can be identified with the Cabibbo angle $\theta_c$ (Cabibbo, 1963). Thus, from equation (78) we have a result that the Weinberg angle is twice the Cabibbo angle, which was already noted (Samiullah, 1986) elsewhere.

In our formalism we are left with two more charged gauge bosons $U^\pm_\mu$ and $V^\pm_\mu$. They transform as spinors; thus, applying the transformation (62) to $U^\pm_\mu$ and $V^\pm_\mu$, the relevant part of $L_{\text{int}}$ can be expressed as

$$\begin{aligned} &\frac{1}{\sqrt{2}} g\tilde{U}^+_\mu(\bar{u}_L\gamma^\mu s_L c - \bar{c}_L\gamma^\mu d_L c + \bar{\nu}_{eL}\gamma^\mu e_L - \bar{\nu}_{\mu L}\gamma^\mu \mu_L) + \text{H.c.} \\ &+\frac{1}{\sqrt{2}} g\tilde{V}^+_\mu(\bar{u}_L\gamma^\mu s_L c + \bar{c}_L\gamma^\mu d_L c + \bar{\nu}_{eL}\gamma^\mu \mu_L + \bar{\nu}_{\mu L}\gamma^\mu e_L) + \text{H.c.} \end{aligned} \tag{79}$$

which is again the expression one would obtain for this part of the $L_{\text{int}}$ by introducing in it the Cabibbo rotation of $d$, $s$, and $\nu_e$, $\nu_\mu$ flavors, respectively, as given in equations (71) and (72).

Thus, in all cases, we have obtained the exact expressions derivable by introducing the Cabibbo rotation of *d*, *s* and $\nu_e$, $\nu_\mu$ flavors.

Recall equation (56), which can be simplified to read

$$P(\nu_{e\theta} \leftrightarrow \nu_{\mu\theta}) = \sin^2 2\theta \sin^2(1.27\,\Delta m^2 R/E) \tag{80}$$

where $\Delta m^2 = m_\mu^2 - m_e^2$ is in units of electron volts squared, $E$ is the neutrino energy in megaelectron volts, and $R$ is the distance from the source of neutrinos in meters. Baker *et al.* (1981) have reported a search for the neutrino oscillation at Fermilab. For $(R/E)_{\text{av}} = 0.16$ m/MeV, they find that the small oscillation probability is given as $P(\mu \to e) = 1.3 \times 10^{-2}$. Inserting these values in (80), we find

$$\sin(2\theta)\,\Delta m^2 \leq 1.6\ \text{eV}^2$$

which for $\theta$ identified with the Cabibbo angle $\theta_c$ sets an upper limit $\Delta m^2 \leq 1.3$ eV$^2$.

## 8. DISCUSSION OF RESULTS

In the present work, on the $O(3)$ sector of our model we have reproduced the usual $SO(3)$ monopole theory together with its triplet of scalars. In view of the several possible choices of Higgs direction for spontaneously breaking the $O(5) \times U(1)$ symmetry down to the $O(3)$ level, the precise direction is of extreme importance, as it has a dual function: (1) to leave the gauge bosons $C_\mu$, $D_\mu$ and $E_\mu$ massless so that they can be used in defining the usual electromagnetic tensor [see equation (60)] and (2) to give a constant value to the Higgs triplet $\bar{\phi}$. From equation (62) it is exactly the angle by which a spinor is to be rotated and is identified with the Cabibbo angle. Equation (60) plays a crucial role, indicating that $W_\mu^i$ has to be rotated in the direction of $\bar{\phi}$, where it acquires a constant value which, to fulfill the consistency demand, results in rotating the different structures such as isodoublets and isotriplets, etc., of the theory appropriately, which in turn Cabibbo-rotates the quarks $d$ and $s$ along with the neutrinos $\nu_e$, $\nu_\mu$. It is only in terms of $\bar{\phi}$ that we have obtained the monopole field [see equation (45)], which shows that the Cabibbo rotation of $u$, $d$ and $\nu_e$, $\nu_\mu$ flavors and the neutrino oscillations are triggered by the existence of a monopole in the theory.

We have two more important by-products of our result, from equation (7.15). The Weinberg angle is twice the Cabibbo angle, which is borne out by the experimental data, and we have arrived at $A_\mu$ by two alternative ways, equations (36) and (74). The former stresses its four-vector aspect, the latter its masslessness as a photon.

## ACKNOWLEDGMENTS

I thank Prof. Abdus Salam, the International Atomic Energy Agency, and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste, where this work was done.

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